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Field Distribution in a Circular Waveguide with a Corrugated Dielectric Lining

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Abstract —**The problem of wave propagation through a circular cylinder with a periodically interrupted dielectric lining is solved by a boundary value approach by considering the region between the corrugations as a medium with a tensor permittivity. The characteristic equation for the phase constant is derived by matching the field components. Solutions for the phase constant are obtained and the variation of the phase constant with the physical parameters is studied. The variation of the axial and circumferential electric field components in the transverse plane is also studied.**

I. Introduction

In many applications involving large reflector antenna systems, there is a growing need for a feed structure that will combine the advantages of high gain, low spillover loss, reduced side-lobe level, low cross-polarization, and high aperture efficiency. Thus, Kay [1] in the U.S. and Minnet and Thomas [2] in Australia independently developed the concepts of a corrugated horn and a corrugated circular waveguide, respectively. In the former case, Kay came to the conclusion that grooved walls in a conical horn would present the same boundary conditions to all polarizations and hence would create a tapered aperture field distribution in all planes, resulting in a symmetric radiation pattern with equal E- and H-plane beam widths. Minnet and Thomas showed that the focal region fields of a paraboloidal reflector consisted of a superposition of cylindrical hybrid modes, which are the natural propagating modes of a circular waveguide with corrugated walls. It was realized that such walls are anisotropic in the sense that they impose the same boundary conditions on the electric and magnetic fields, which in turn would lead to an axially symmetric radiation pattern. Clarricoats and Saha [3] carried out a detailed analysis of the propagation and radiation characteristics of a corrugated circular waveguide feed. The radiation pattern and cross-polarization of a dielectric-lined circular waveguide feed were determined by Kumar [4]. If the dielectric lining of the circular waveguide is periodically interrupted, it is believed that the cross-polarization will be significantly reduced, and some preliminary calculations were reported by Mahmoud and Aly [5]. In their study, the region between two disks is considered a medium with tensor permittivity. In the present article, we study the boundary value problem of a dielectric-disk loaded circular waveguide and investigate the propagation characteristics, for example, the phase constant and its variation with the physical parameters of the structure. More details are given elsewhere [6]. Furthermore, the field distribution in the transverse plane is studied; in particular, the variation of the axial and circumferential electric field components with the normalized radius is examined.

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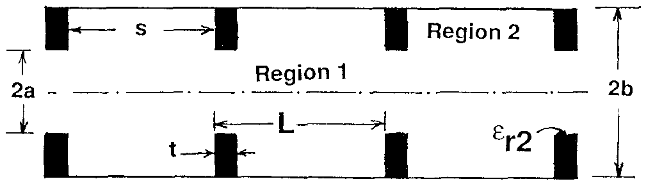

Fig. 1. Geometry of the problem.

II. Solution of the Boundary Value Problem

The geometry of the structure under investigation is shown in Fig. 1. A circular waveguide with an internal diameter of $2b$ exists with its axis coinciding with the z axis of the cylindrical coordinate system (ρ, ϕ, z). The walls of the waveguide are assumed to be perfectly conducting and the waveguide is periodically loaded with dielectric disk of internal diameter $2a$ and external diameter $2b$. The disks have a thickness t and the interdisk spacing is assumed to be s. The relative dielectric constant of the disks is ϵ_{r2} and for generality we assume the region $0 \leqslant \rho \leqslant a$ to have a dielectric constant ϵ_{r1}. The region between $\rho = a$ and $\rho = b$ is assumed to have a tensor permittivity whose components are given by [5]

$$\epsilon = \begin{bmatrix} \epsilon_t & 0 & 0 \\ 0 & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \tag{1a}$$

where

$$\epsilon_z = \epsilon_0 \epsilon_{r2} / [\epsilon_{r2} - (t/L)(\epsilon_{r2} - 1)] \tag{1b}$$

and

$$\epsilon_t = \epsilon_0 [1 + (\epsilon_{r2} - 1)(t/L)], \qquad L = t + s. \tag{1c}$$

The axial components of the electric and magnetic fields in region 1 $(0 \leqslant \rho \leqslant a)$ are given by

$$E_{z1} = A_1 J_1(k_1 \rho) \cos\phi \exp(-j\beta z) \tag{2a}$$

$$\eta_0 H_{z1} = B_1 J_1(k_1 \rho) \sin\phi \exp(-j\beta z) \tag{2b}$$

where A_1 and B_1 are the amplitude constants, $J_1(k_1\rho)$ is the Bessel function of the first kind and order 1, and β is the axial phase constant. The transverse wavenumber is given by $k_1 = (k_0^2 \epsilon_{r1} - \beta^2)^{1/2}$, where k_0 is the free-space wavenumber $(\omega\sqrt{\mu_0 \epsilon_0})$. In a similar manner, the axial components of the electric and magnetic fields in region 2 $(a \leqslant \rho \leqslant b)$ can be written as

$$E_{z2} = A_2 P_1(k_{2M}\rho)\cos\phi \exp(-j\beta z) \quad (3a)$$

$$\eta_0 H_{z2} = B_2 Q_1(k_{2E}\rho)\sin\phi \exp(-j\beta z) \quad (3b)$$

where A_2 and B_2 are the amplitude coefficients and the transverse wavenumbers k_{2E} and k_{2M} are given by

$$k_{2E}^2 = k_0^2(\epsilon_t/\epsilon_0) - \beta^2 \quad (4a)$$

$$k_{2M}^2 = (\epsilon_z/\epsilon_t)k_{2E}^2. \quad (4b)$$

The electric and magnetic fields in region 2 have different transverse wavenumbers, k_{2M} and k_{2E}, and this is a characteristic of the tensor medium. The functions $P_1(k_{2M}\rho)$ and $Q_1(k_{2E}\rho)$ must satisfy the boundary conditions at the conducting boundary at $\rho = b$ and are hence given by

$$P_1(k_{2M}\rho) = [J_1(k_{2M}\rho)Y_1(k_{2M}b) - J_1(k_{2M}b)Y_1(k_{2M}\rho)]/Y_1(k_{2M}b) \quad (5a)$$

$$Q_1(k_{2E}\rho) = [J_1(k_{2E}\rho)Y_1'(k_{2E}b) - J_1'(k_{2E}b)Y_1(k_{2E}\rho)]/Y_1'(k_{2E}b). \quad (5b)$$

Once the axial components of the electric and magnetic fields are known, the circumferential components in each region can be calculated from Maxwell's equations. They are given in region 1 by

$$E_{\phi 1} = j\sin\phi[(\beta/k_1)A_1\{J_1(k_1\rho)/k_1\rho\} + (k_0/k_1)B_1 J_1'(k_1\rho)] \quad (6a)$$

$$\eta_0 H_{\phi 1} = -j\cos\phi[A_1(k_0/k_1)\epsilon_{r1}J_1'(k_1\rho) + B_1(\beta/k_1)\{J_1(k_1\rho)/k_1\rho\}] \quad (6b)$$

and in region 2 by

$$E_{\phi 2} = j\sin\phi[(\beta/k_{2E})A_2\{P_1(k_{2M}\rho)/k_{2M}\rho\} + B_2(k_0/k_{2E})Q_1'(k_{2E}\rho)] \quad (7a)$$

$$\eta_0 H_{\phi 2} = -j\cos\phi[(k_0/k_{2E})(\epsilon_t/\epsilon_0)A_2(k_{2M}/k_{2E})P_1'(k_{2M}\rho) + B_2(\beta/k_{2E})\{Q_1(k_{2E}\rho)/k_{2E}\rho\}] \quad (7b)$$

where

$$P_1'(k_{2M}\rho) = [J_1'(k_{2M}\rho)Y_1(k_{2M}b) - J_1(k_{2M}b)Y_1'(k_{2M}\rho)]/Y_1(k_{2M}b) \quad (8a)$$

$$Q_1'(k_{2E}\rho) = [J_1'(k_{2E}\rho)Y_1'(k_{2E}b) - J_1'(k_{2E}b)Y_1'(k_{2E}\rho)]/Y_1'(k_{2E}b). \quad (8b)$$

The boundary conditions at $\rho = a$ require the continuity of the tangential electric and magnetic field components, and the elimination of the constants leads to the following characteristic equation:

$$\left[\frac{(\epsilon_z/\epsilon_0)}{k_{2M}a}\frac{P_1'(k_{2M}a)}{P_1(k_{2M}a)} - \frac{\epsilon_{r1}}{k_1 a}\frac{J_1'(k_1 a)}{J_1(k_1 a)}\right] \cdot \left[\frac{1}{k_{2E}a}\frac{Q_1'(k_{2E}a)}{Q_1(k_{2E}a)} - \frac{1}{k_1 a}\frac{J_1'(k_1 a)}{J_1(k_1 a)}\right] = (\beta/k_0)^2[(k_1 a)^{-2} - (k_{2E}a)^{-2}]^2. \quad (9)$$

The other equation relating the two transverse wavenumbers k_1 and k_{2E} is given by

$$k_{2E}^2 - k_1^2 = k_0^2\{(\epsilon_t/\epsilon_0) - \epsilon_{r1}\} \quad (10)$$

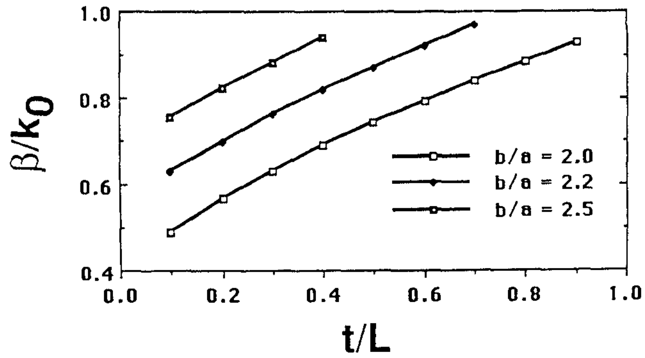

Fig. 2. Variation of β/k_0 with t/L and b/a; $k_0 a = 1.0$, $\epsilon_{r2} = 2.56$.

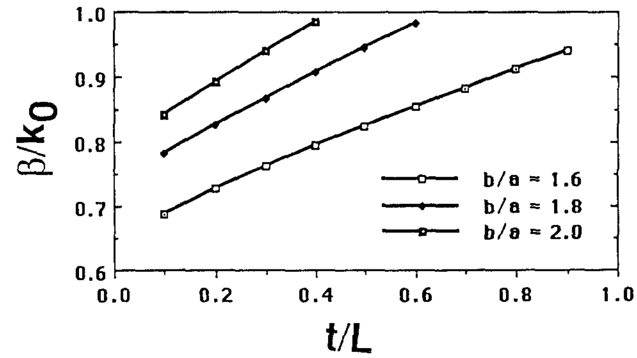

Fig. 3. Variation of β/k_0 with t/L and b/a; $k_0 a = 1.5$, $\epsilon_{r2} = 2.56$.

The two equations (9) and (10) are solved to obtain k_1 and k_{2E}. Once k_{2E} is known, the other propagation constant, k_{2M}, can be easily determined. The normalized phase constant (β/k_0) can be calculated from the equation

$$(\beta/k_0)^2 = (\epsilon_t/\epsilon_0) - (k_{2E}/k_0)^2. \quad (11)$$

The amplitudes are obtained from the boundary conditions and the relations

$$(A_1/B_1)^2 = [(1/k_{2E}a)\{Q_1'(k_{2E}a)/Q_1(k_{2E}a)\} - (1/k_1 a)\{J_1'(k_1 a)/J_1(k_1 a)\}] \cdot [(\epsilon_z/\epsilon_0)(1/k_{2M}a)\{P_1'(k_{2M}a)/P_1(k_{2M}a)\} - (\epsilon_{r1}/k_1 a)\{J_1'(k_1 a)/J_1(k_1 a)\}]^{-1} \quad (12)$$

$$A_2/A_1 = J_1(k_1 a)/P_1(k_{2M}a) \quad (13)$$

$$B_2/B_1 = J_1(k_1 a)/Q_1(k_{2E}a). \quad (14)$$

III. Results and Discussion

The characteristic equation is numerically solved for a given set of parameters and some of the results for the normalized phase constant are shown in Figs. 2 and 3. Fig. 2 shows the variation of the normalized phase constant (β/k_0) with the ratio t/L for three values of the ratio b/a for a fixed value of $k_0 a = 1.0$. In general, the phase constant increases with increasing thickness ratio and b/a. In other words, closer spacing of the dielectric disks in a circular waveguide of constant diameter increases the value of the phase constant or decreases the phase velocity of the wave compared with a wider spacing. The parameter b/a may also be viewed as the corrugation depth. For the

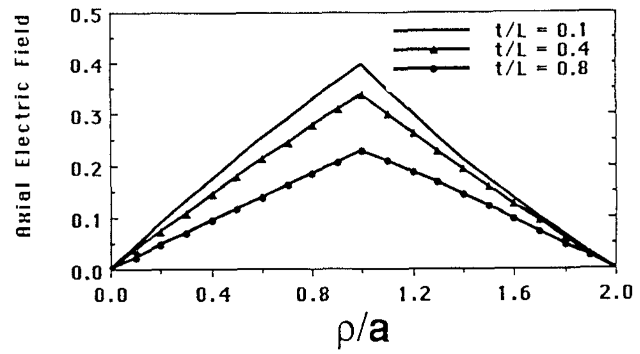

Fig. 4. Variation of the axial electric field with ρ/a and t/L; $b/a = 2.0$, $\epsilon_{r2} = 2.56$.

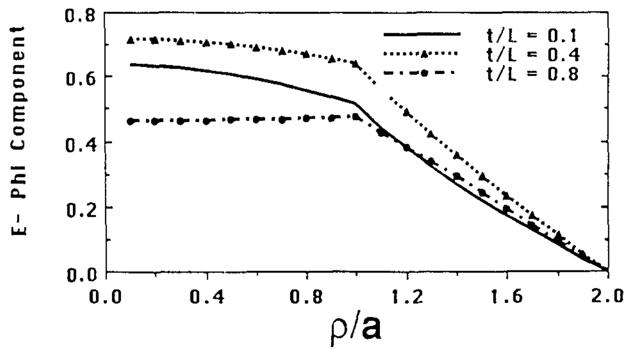

Fig. 5. Variation of the circumferential electric field with ρ/a and t/L; $b/a = 2.0$, $\epsilon_{r2} = 2.56$.

same t/L ratio and k_0a, the normalized phase constant increases with increasing corrugation depth. Fig. 3 shows the variation of β/k_0 with t/L for a larger value of k_0a. Increasing k_0a appears to result in a larger value of β/k_0 for the same t/L ratio. When all the physical parameters and the permittivity remain unchanged, increasing k_0a corresponds to an increase in frequency. Hence, an increase in frequency acts to slow down the propagating wave in a cylindrical waveguide loaded with dielectric disks. Consequently, the ratio β/k_0 approaches unity for a smaller value if k_0a is larger. We can also view the disk-loaded cylindrical waveguide as a band-pass filter and these results indicate that an increase in frequency would lead to a narrowing of the bandwidth.

Fig. 4 shows the variation of the axial component E_z with the normalized radius for three different values of the thickness ratio. The axial electric field is zero on the axis, increases slowly in region 1 to a maximum value, decreases in region 2, and finally vanishes at the conducting boundary at $\rho = b$. The magnitude of the electric field decreases with increasing t/L ratio.

Fig. 5 shows the variation of the circumferential electric field component in the transverse plane. The ϕ component starts at a constant value on the axis and is almost constant in region 1 for larger t/L ratios. For lower values of t/L, however, it tends to decrease slightly. In region 2, it decreases monotonically and at the conducting boundary it goes to zero. Once again, we notice that the magnitude is higher for higher values of the thickness ratio. The variations of the axial and circumferential components of the electric field with respect to the angular coordinate ϕ are given by $\cos\phi$ and $\sin\phi$, respectively.

IV. Conclusions

The problem of guided wave propagation in a circular waveguide with a corrugated dielectric lining is solved by a boundary value problem by regarding the medium between the corrugations as a medium with tensor permittivity. By matching the field components at the boundaries, the characteristic equation for the phase constant is derived. This characteristic equation, which is transcendental in nature, is numerically solved for a given set of physical parameters, for example the diameter of the cylinder, the corrugation depth, the thickness ratio, and the permittivity of the dielectric material. Some representative results of the normalized phase constant and its variation with different parameters are shown. The variations of the axial and circumferential electric field components in the transverse plane with the normalized radius are shown for different values of the thickness ratio.

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Asymmetrical Coplanar Waveguide with Finite Metallization Thickness Containing Anisotropic Media

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Abstract—**The spectral-domain approach (SDA) is extended in the present paper for symmetrical and asymmetrical coplanar waveguides with anisotropic media. The quasi-static and the hybrid-mode analytical method are developed in the spectral domain taking the metallization thickness effect into consideration. Numerical computations include the quasi-static and frequency-dependent hybrid-mode values of the phase constants and characteristic impedances for the symmetrical and asym-**

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